Bresenham Line Drawing Algorithm.

## **Theory**

Bresenham's line algorithm is an accurate and efficient line drawing algorithm. It uses only integer arithmetic to find the next position to be plotted. It avoids incremental error. The major concept of Bresenham's algorithm is to determine the nearest pixel position. Great advantage of this algorithm is that it can be used to display circles and other curves. In Bresenham's algorithm, we calculate the decision parameter which decides which pixel to select and which function is used for next decision parameter.

For positive slope |m|<1

Pixel positions are determined by sampling at unit x intervals. Starting from left end position (x0, y0) of a given line, we step to each successive column (x-position) and plot the pixel whose scan line y value is closed to the line path. Assuming the pixel at (x, y) to be displayed is determined, we next to decide which pixel to plot in column X+1, our choices are the pixels at positions.

(xk+1, yk) and (xk + 1, yk + 1)

At sampling position xk + 1. we label vertical pixel separations from the mathematical line path d1 and d2. The y-co-ordinate on the mathematical at pixel column position xk + 1 is calculation.

y=m(xk+1)+b

Then.

d₁=y-yk

d₁=m(xk+1)+b-yk

And,

d2 =(yk+1)-y

d2=(yk+1)-m(xk+1)-b

Now,

d1-d₂=m(xk+1)+b-ук-ук -1+m(xk+1)+b

=2m(xk+1)-2yk+2b-1

=2(x+1)-2yk+2b-1

Defining decision parameter pk = Δx(d₁-d2)

Pk =Δx(d1-d₂)=2Δy(xk+1)-2Δxyk+2Δxb - Δx

= 2Δухk +2Δу - 2Δхуk+Δx(2b-1)

= 2Δух-2Δхуk+2Δy+Δx(2b-1)

= 2Δухk-20хуk+c

Case I: If pk < 0 then d1 < d2 which implies pixel at yk is nearer than pixel at (yk + 1). So, pixel at yk is better to choose which reduce error than pixel at (yk + 1). This determines next pixel co-ordinate to plot is (xk + 1, yk)

Case II: If pk ≥ 0, then d2 < d1, which implies that yk + 1 is nearer than yk. So, pixel at (yk + 1) is better to choose which reduce error than pixel at yk. This determines next pixel co-ordinate to plot is (xk + 1, yk + 1).

Now, similarly, pixel as (xk + 2) can be determined whether it is (xk + 2, yk + 1) or (xk + 2, yk +2) by looking the sign of deciding parameter pk+1 assuming pixel as (xk+1) is known

Pk+1 =2Δухk+1-2Δхуk+!+c

Pk+1 - Pk =2Δу-2Δх(yk+1-yk)

This implies that decision parameter for the current column can be determined if the decision parameter of the last column is known.

Here, (yk+1–yk) could either 0 or 1 which depends on sign of pk.

If p­k≥ 0 (i.e.,d2< d1), yk+1= yk + 1 which implies (yk + 1 – yk) = 1

That is, at pk≥0, the pixel to plot is (xk + 1, yk + 1) and pk+1 = pk + 2∆y – 2∆x

If p­k< 0 (i.e.,d2> d1), yk+1= yk which implies (yk – yk) = 0

That is, at pk<0, the pixel to plot is (xk + 1, yk + 1) and pk+1 = pk + 2∆y

For positive slope |m|>1

Pixel positions are determined by sampling at unit y intervals. Starting from left end position (x0, y0) of a given line, we step to each successive row (y-position) and plot the pixel whose scan line x value is closed to the line path.

Assuming the pixel at (x, y) to be displayed is determined, we next to decide which pixel to plot in rowy+1, our choices are the pixels at positions.

(Xk, yk+1) and (xk + 1, yk+1)

At sampling position yk + 1, we label horizontal pixel separations from the mathematical line path d1 and d₂

The x-co-ordinate on the mathematical at pixel row position yk + 1 is calculation.

As,

Yk+1 = mx +b

x=(yk+1-b)/m

Then,

d₁ = x-xk

And,

d₂=(xk+1)- x

Now,

d1-d2 =2x-2xk-1

x=(yk+1-b)/m

d1-d2=2(yk+1-b)/m-2xk-1

=2Δx (yk+1-b)/ Δy-2xk-1

Δy (d1-d₂)=2Ax yk+2Δx-2Δx b-2Δу хk- Δу

Defining decision parameter pk = Δy (d1-d₂)

Pk=Δy (d1-d₂) = 2Δx yk+2Δx-2Δx b-2Δy xk- Δy = 2Δх уk-2Δу хk+с

Case I: If pk < 0 then d1 < d2 which implies pixel at xk is nearer than pixel at (xk + 1). So, pixel at xk is better to choose which reduce error than pixel at (xk + 1). This determines next pixel co-ordinate to plot is (xk, yk+1)

Case II: If pk ≥ 0, then d1> d2, which implies that xk + 1 is nearer than xk. So, pixel at (xk + 1) is better to choose which reduce error than pixel at xk. This determines next pixel co-ordinate to plot is (xk + 1, yk + 1). Now, similarly, pixel as (yk + 2) can be determined whether it is (xk + 1, yk + 2) or (xk + 2, yk +2) by looking the sign of deciding parameter pk+1 assuming pixel as (yk+1) is known.

Pk+1= 2∆x yk+1– 2∆y xk+1 +c where c is same as in pk

Pk+1-Pk=2Δx-2Δy(xk+1-xk)

This implies that decision parameter for the current row can be determined if the decision parameter of the last row is known. Here, (xk+1 – xk) could either 0 or 1 which depends on sign of pk.

If pk < 0(i.e.d1 < d2), xk+1 = xk which implies (xk+1−xk = 0) i.e.,

the pixel to plot is (xk, yk + 1) and pk+1 = pk + 2∆x

If pk ≥0(i.e.d1 > d2), xk+1= xk + 1 which implies (xk + 1 – xk) = 1

That is, at pk≥0, the pixel to plot is (xk + 1, yk + 1) and pk+1 = pk + 2∆x – 2∆y

## **BLA Algorithm**

Step 1. Start

Step 2. Declare variables x1,y1,x2,y2,1,ly, Δx, Δy,p0, pk,pk+1

Step 3. Read values of x1, y1,x2,y2

Step 4. Calculate Δx = absolute(x2-X1)

Δy = absolute(y2-y1)

Step 5. If (x2>x₁)

assign 1x= 1

else

assign 1x = -1

Step 6. if (y2>y1)

assign 1y = 1

else

assign 1y = -1

Step 7. Plot (x1, y1)

Step 8. if x>Δy (i.е., m<1)

compute p0 = 2Δy-Δx

starting at k = 0 to Δx times, repeat

if (pk<0)

Xk+1=X+1x

Уk+1=Ук

Pk+1=Pk+2Δy

else

Xk+1=Xk+1x

Уk+1=Ук+1y

Pk+1=Pk+2Δy-2Δx

plot(xk+1, Yk+1)

else

calculate p = 2Δх-Δу

starting at k = 0 to Δy times, repeat

if(p<0)

Xk+1=Xk

Уk+1=Ук+1y

Pk+1=Pk+2Δx

else

Xk+1=Xk

Yk+1=Yk+ly

PK+1=Pk+2Δx-2Δу

plot(Xk+1,Yy+1)

Step 9. Stop